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Third Year Bachelor of Science (Sem. - VI)

Examination September - 2025

Mathematics (Paper : MTH-603) Real Analysis - III

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **Third Year Bachelor of Science (Sem. - VI)**

Name of the Subject :

☛ **Mathematics (Paper : MTH-603) Real Analysis - III**

Subject Code No.: 2203000206020033

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the questions.

(4) Follow usual notations.

Q. 1. Answer the following questions (Any Five): (10)

1. Is a sequence $\{n^3\}_{n=1}^{n=\infty}$ $(C, 1)$ summable? Justify your answer.
2. Give an example of a sequence which is $(C, 2)$ summable but not $(C, 1)$ summable.
3. Does $\left\{\frac{nx}{1+(nx)^2}\right\}_{n=1}^{\infty}$, $-\infty < x < \infty$ converge uniformly? Justify your answer.
4. Show that empty set is of measure zero.
5. If $f \in R[a, b]$ and $\lambda \in R$, then prove that $\int_a^b -f = -\int_a^b f$.
6. Let $\tau = \left\{0, \frac{3}{4}, 1\right\}$ be a subdivision of $[0, 1]$ then find the refinement of τ.

7. Evaluate: $\lim_{n\to\infty}\left[\frac{1}{n+2}+\frac{1}{n+4}+\frac{1}{n+6}+\frac{1}{n+8}+\dots\dots+\frac{1}{n+2n}\right]$.

8. If $f(x)=\int_0^x \sqrt{t+t^7}\,dt\ (x>0)$, then find $f'(6)$.

Q. 2. Answer the following (Any two): (10)

a. Prove that the sequence –1, 2, 2, –1, 2, 2, –1, 2, 2 ... is $(C, 1)$ summable

b. If $\sum_{n=1}^{\infty} a_n$ is $(C, 1)$ summable, then prove that a sequence $\{S_n\}_{n=1}^{\infty}$ is also $(C, 1)$ summable.

c. Let $\{S_n\}_{n=1}^{n=\infty}$ be a monotone sequence. Then prove that $\{\sigma_n\}_{n=1}^{n=\infty}$ is a monotone sequence; where $\sigma_n = \dfrac{S_1+S_2+S_3+\dots\dots+Sn}{n}$

Q. 3. Answer the following (Any two): (10)

a. Let $\{f_n\}_{n=1}^{n=\infty}$ be a sequence of real valued functions in $R[a, b]$; which converges uniformly to the function f on $[a, b]$. Then prove that $\lim_{n\to\infty}\int_a^b f_n(x)=\int_a^b f(x)$

b. State and prove Cauchy criterion for uniform convergence of sequence of functions.

c. Let $\{f_n\}_{n=1}^{n=\infty}$ be a sequence of real valued functions in $R[a, b]$; which converges uniformly to the function f on $[a, b]$. If for each $n \in I$, $F_n(x)=\int_a^x f_n(t)\,dt$, then prove that $\{f_n\}_{n=1}^{n=\infty}$ converges uniformly.

Q. 4. Answer the following (Any two): (10)

a. If $f \in R[a, b]$ and $a < c < b$, then prove that $f \in R[a, c], f \in R[c, b]$ and $\int_a^b f=\int_a^c f+\int_c^b f$.

b. Let $f(x)=x,\ 0\le x\le 1$ and let $\sigma=\left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$ be a subdivision of $[0, 1]$. Then find $U[f, \sigma]$ and $L[f, \sigma]$.

c. If $f \in R[a, b]$ and $f(x)\ge 0$ is at almost every point in $[a, b]$, then prove that $\int_a^b f \ge 0$.

Q. 5. Answer the following (Any two). (10)

a. If f' and g' are continuous on $[a, b]$, then prove that

$\int_a^b f(x)\, g'(x)\, dx = f(b) g(b) - f(a) g(a) - \int_a^b f'(x) g(x)\, dx.$

b. Let Ø be a real valued function on the closed bounded interval $[a, b]$ such that Ø$'$ is continuous on $[a, b]$. Let $Ø(a) = A$, $Ø(b) = B$. If f is continuous on $Ø([a, b])$ then prove that $\int_A^B f(x)\, dx = \int_a^b f[Ø(u)]\, Ø'(u) du$.

c. If $f \in R[a, b]$ and if $F(x) = \int_a^b f(t)\, dt$, then prove that

$F(b) - F(a) = \int_a^b f(x)\, dx.$
